Propositions, Truth and Paradox
PHIL 428/628, Week 9: April 12 2016

FIELD

Two relatively independent strands in the paper: issues concerning ‘intersubstitutivity’ and the truth schema; and issues concerning revenge (including issues relating to the notion of determinacy).

1. Intersubstitutivity and the Truth Schema

The truth schema: $T(\langle A\rangle) \leftrightarrow A$ (for any sentence A, where Field uses $\langle A\rangle$ for the ‘standard name’ of A; e.g., a numeral for the gödel number of A under some system of such numbering).

Intersubstitutivity: ‘If Y results from X by replacing some occurrences of A with True($\langle A\rangle$), then X and Y entail each other.’ (What does ‘entail’ mean here? Presumably: follows from the proposed logic together with the proposed principles and rules governing ‘True’.)

Intersubstitutivity entails the truth schema if $A \leftrightarrow A$ is a logical truth.

The strong Kleene version of Kripke has intersubstitutivity but not the truth schema ($A \leftrightarrow A$ is not a truth of strong Kleene logic). The supervaluationist version does not have intersubstitutivity or the truth schema (since if Q is a Liar sentence then $Q \vee \neg Q$ will be true, but True($\langle Q\rangle$) $\vee$ True($\langle \neg Q\rangle$) will not be).

Why is intersubstitutivity important? E.g., want equivalence of:

- If everything Jones said is true then ___
- If A_1 and … and A_n then ___

given assumption that Jones said exactly $A_1, \ldots, A_n$.

It is not so clear why the truth schema in and of itself is so desirable—at least not from anything Field says in this paper. Nor is it clear from anything he says in this paper what he has against the strong Kleene version of Kripke. But I think his main objections are: (a) the logic is too weak (e.g., no logical truths at all), and so cannot vindicate ‘ordinary reasoning’; and (b) we cannot in that construction say of the Liar sentence that it is ‘defective’.

The logic that Field proposes is strong Kleene logic plus a new conditional (i.e., $P \rightarrow Q$ is not defined as $\neg P \vee Q$).

To get what Field wants it seems that we will need to move beyond 3-valued logic and semantics (i.e., logic and semantics with values t, f and u; or 1, 0 and 1/2).

- Because suppose Q is a Liar sentence $\neg T(c)$. This will receive value 1/2. So will $T(\langle Q\rangle)$.
- Field wants the truth schema, so $T(\langle Q\rangle) \leftrightarrow Q$ must be true.

- Thus, the biconditional P ↔ R will have to be true whenever P and R have value 1/2 (assuming compositionality).
- But then (P ↔ ¬P) ∨ ¬P will be true precisely when P gets value either 0 or 1/2.
- I.e., we have defined exclusion negation and will then get all the problems that entails.

Thus, Field replaces a single third value with many more. Indeed, the cardinality of the set of values will depend on the cardinality of the model one starts with. The conditional-free fragment of the resulting language will essentially be just the strong Kleene version of Kripke.

The basic idea is that the values will be partially ordered, and P → R will be true (i.e., receive value 1) iff the value of P ≤ the value of R.

2. Revenge

But this does not seem *terribly* intimately related to the other aspect of Field's paper: which is his claim that the sort of solution he proposes is 'revenge immune'. That is, it seems to me that most of the anti-revenge arguments that he offers on behalf of his solution could just as well be offered by a solution the technical component of which was simply strong Kleene Kripke.

On the face of it, Field faces a revenge problem just as Kripke did.

- Kripke faced a problem with a logical operator (form of negation?) ~ with ~P getting value 1 iff P gets value ≠ 1.
- For consider ~T(c) where c denotes this sentence. ~T(c) and T(c) cannot get the same value. Thus, we're in trouble if we want T(⟨A⟩) and A to always get the same value (where ⟨A⟩ is *any* name of A).
- Indeed, we're in trouble even if all we want is T(⟨A⟩) to get 1 iff A does.
- For if ~T(c) gets 1, then T(c) must not get 1; and if ~T(c) doesn't get 1 then T(c) must get 1.
- But Field seems to face just the same problem!
- Going from 1/2 to an infinite set of new values seems irrelevant.

How does Field respond?

- These constructions with sets of values etc. are just models. They are constructions in standard, classical set theory, and they serve a merely instrumental role: allowing the definition of validity in the language, and consequently to specifying the theory of truth.
- I take it that someone offering strong Kleene Kripke could respond in just the same way. Field doesn't like their definition of validity (he wants his conditional too), but qua revenge-immunity there doesn't seem to be any difference. But that is not to say the response doesn't work!

How does this response help?

- The hope is that it blocks an argument for the coherence of this new operator ~.

Yet can't we argue for the coherence of ~ in another way? How does Field argue for the coherence of his conditional?

- He states a bunch of basic principles and rules, and shows that they are consistent by producing models. This is supposed to be enough to establish the coherence of his conditional.
- But can't the proponent of ~ do just the same thing?
- This will validate the inference: $P \leftrightarrow \sim P \vDash \bot$ (among others).
- The addition of this operator to any theory that has one of Field's models will be perfectly consistent (indeed it will be a conservative extension of that theory).
- Doesn't this show that ~ governed by just the rules one would expect is perfectly coherent?

But Field's approach won't work with such an operator in the language—any more than Kripke's construction works if exclusion negation is in the language.

- Let Q be a sentence with: $Q \leftrightarrow \sim T(\langle Q \rangle)$
- The truth schema gives: $T(\langle Q \rangle) \leftrightarrow \sim T(\langle Q \rangle)$.
- This gives: $\bot$.

What is gained by denying that ~ is coherent?